Nonleptonic B Decays in SCET (quasi 2-body & 3-body)

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Three-Body Charmless B-decay Workshop
LPNHE, Feb. 2006

Outline

power expansion of QCD

- Nonleptonic decays & Soft-Collinear Effective Theory (SCET)

  i) Factorization Theorem (formal issues)

  ii) Applying the result (phenomenological choices)

- Applications

  i) $B \to \pi\pi$ $\quad B \to K\pi\,, K\bar{K}$ $\quad$ isosinglets

  ii) comments on $B \to VV, B \to VP$

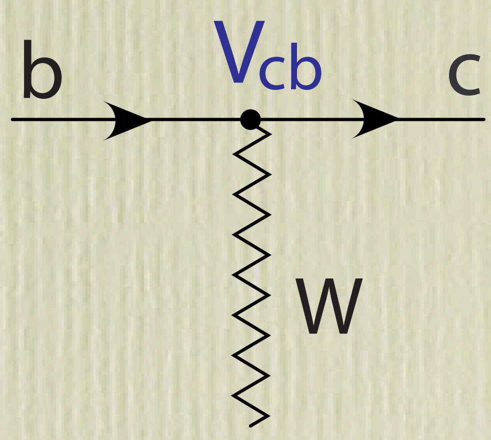
  iii) comments on 3-body decays

B decays - Motivation

- Probe the flavor sector of the SM

CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



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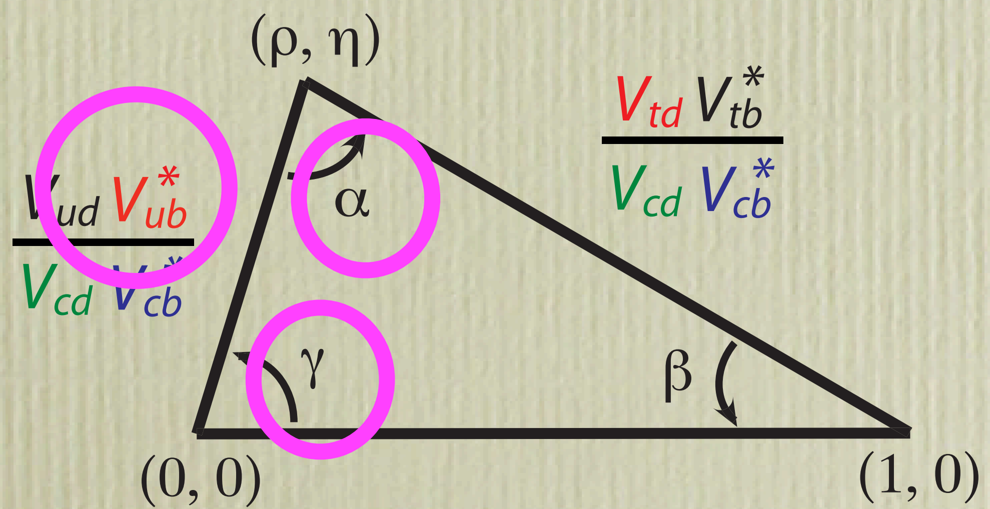
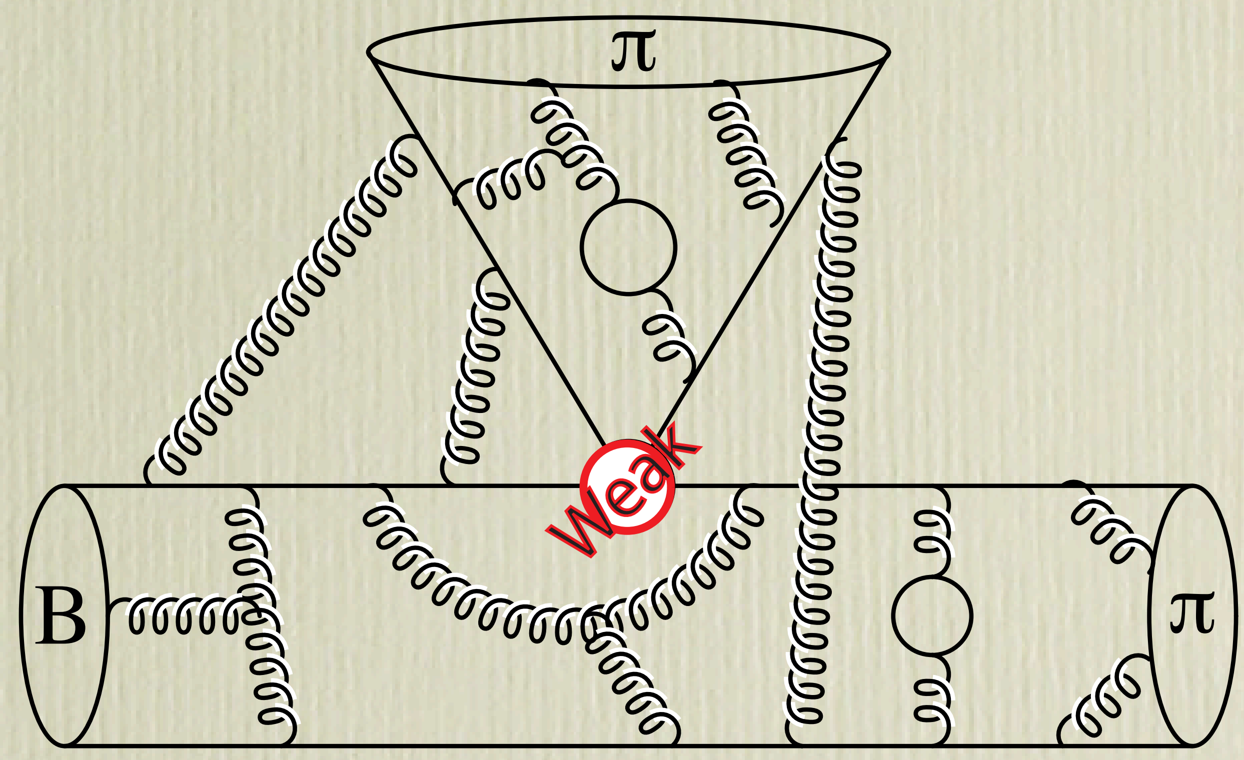
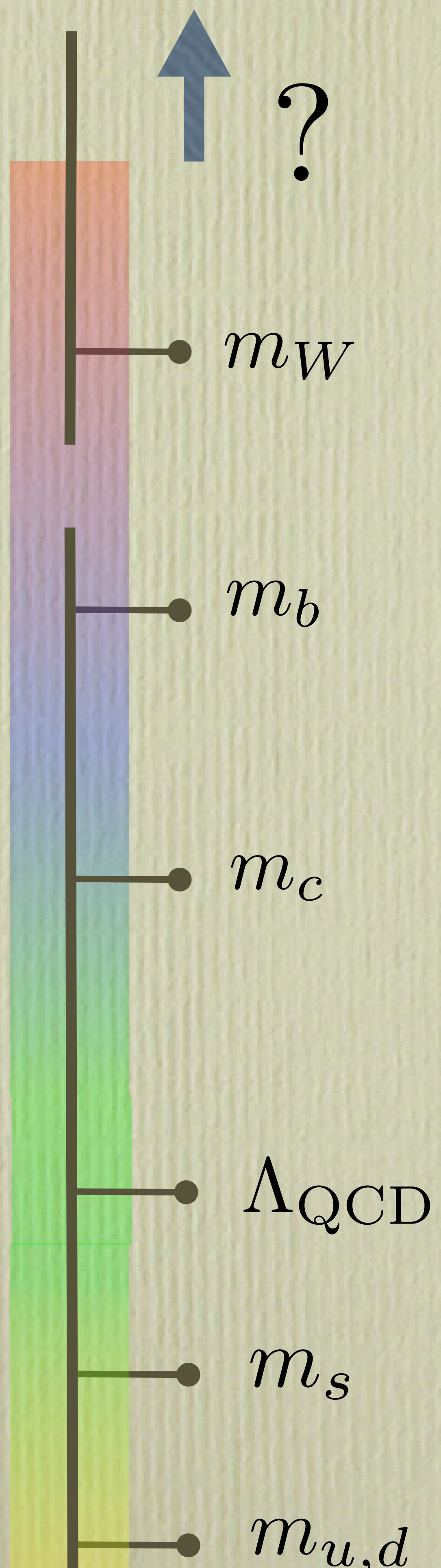




Model Independent Expansions



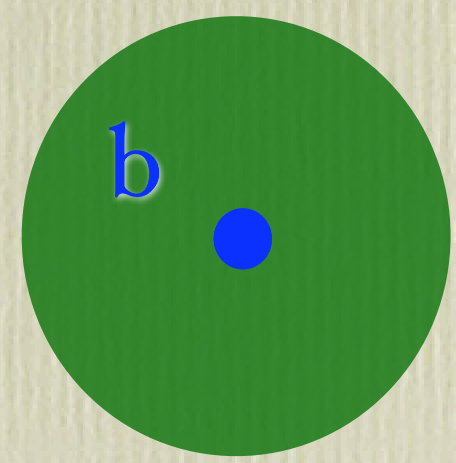
- $m_W, m_t \gg m_b$

$C_1 > C_2, C_{7\gamma}, C_{8g} \gg C_{4,6} > C_{3,5,9,10} > C_{7,8}$

$$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$$

- $m_b \gg \Lambda_{QCD}$

B-meson

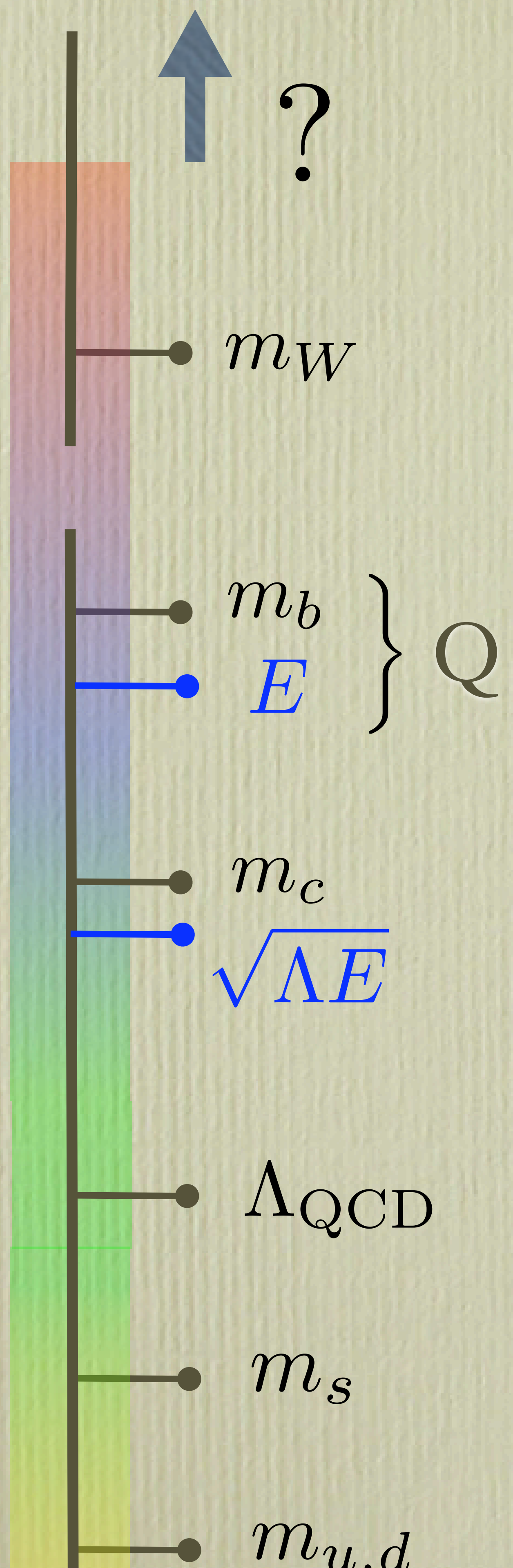


Heavy Quark Effective Theory

h_v, q

- $\Lambda \gg m_{s,d,u}$ SU(3)
- $\Lambda \gg m_{d,u}$ SU(2)

Model Independent Expansions



- $E_\pi \gg \Lambda_{\rm QCD}$ Energetic Hadrons

$B \to M_1 M_2$

Factorization Theorems

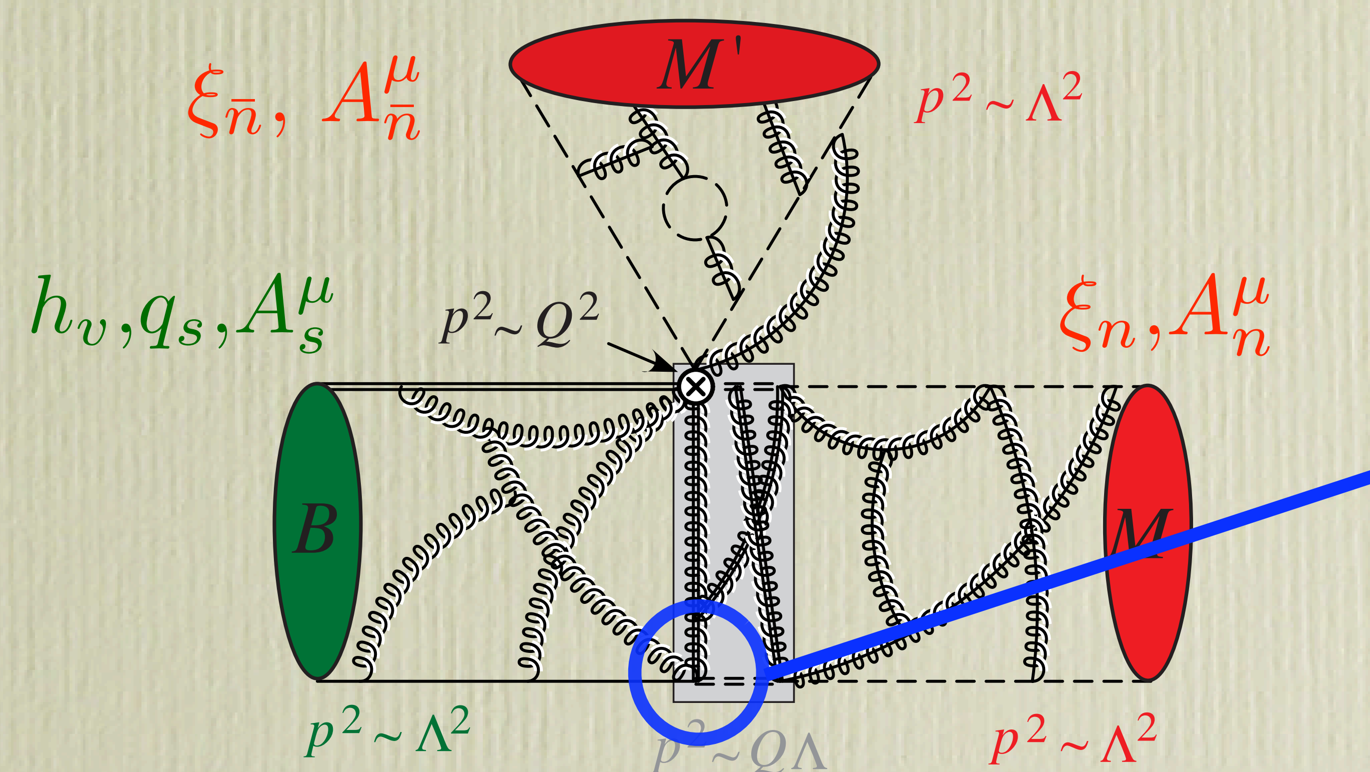
$$A = \int dz dx_i dk^+ T(z)\, J(z, x_i, k^+)\, \phi_1(x_1)\phi_2(x_2)\phi_B(k^+) + \ldots$$

$$Q^2 \gg E\Lambda \gg \Lambda^2$$

Soft-Collinear Effective Theory (SCET)

Bauer, Pirjol, I.S.
Fleming, Luke

many other authors



Decay starts at subleading order

$B \to M_1 M_2$ Factorization (with SCET)

Bauer, Pirjol, Rothstein, I.S.

Operators

QCD

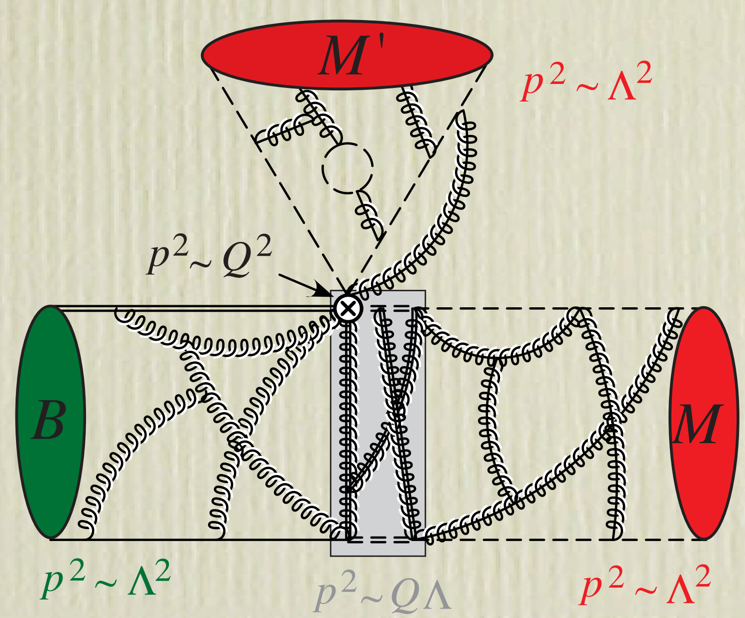
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \Big(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,8g} C_i O_i \Big)$$

SCET$_\mathrm{I}$ Integrate out $\sim m_b$ fluctuations

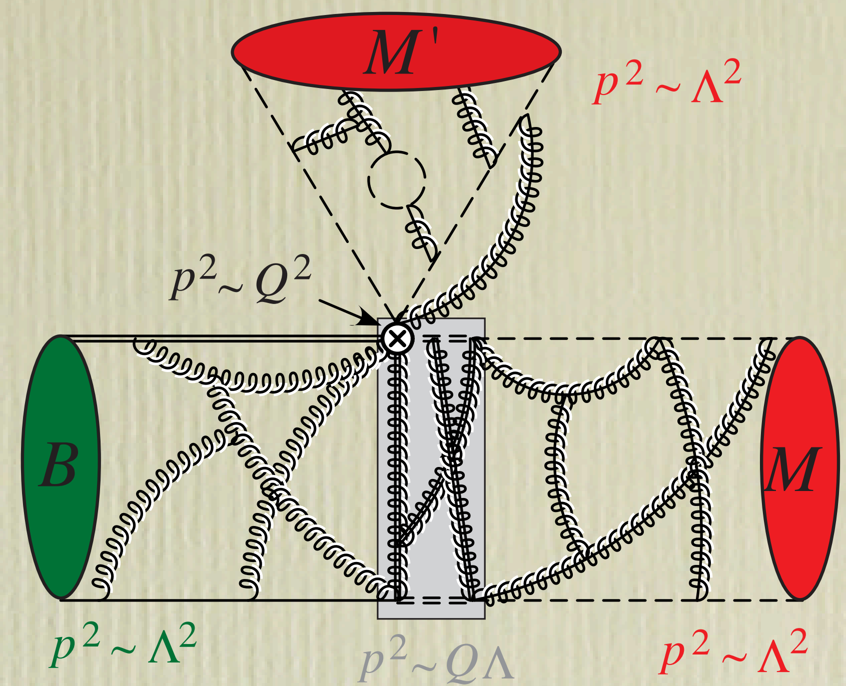
$$H_W = \frac{2G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{6} \int d\omega_j \, c_i^{(f)}(\omega_j) Q_{if}^{(0)}(\omega_j) + \sum_{i=1}^{8} \int d\omega_j \, b_i^{(f)}(\omega_j) Q_{if}^{(1)}(\omega_j) + \mathcal{Q}_{c\bar{c}} + \ldots \Big\}$$

$$Q_{1d}^{(0)} = \big[\bar{u}_{n,\omega_1} \bar{n}\!\!\!/ P_L b_v\big] \big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big] \,, \quad \ldots$$

$$Q_{1d}^{(1)} = \frac{-2}{m_b} \big[\bar{u}_{n,\omega_1} \, ig \mathcal{B}\!\!\!/^{\perp}_{n,\omega_4} P_L b_v\big] \big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big] \,, \quad \ldots$$



Factorization at m_b



Nonleptonic $B \to M_1 M_2$

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\left\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$$

Form Factors

$B \to$ pseudoscalar: $f_+,\ f_0,\ f_T$
$B \to$ vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E)$$
} "hard spectator", "factorizable"

$$+\ C(E)\, \zeta^{BM}(E)$$
} "soft form factor", "non-factorizable"

→ universality at $E\Lambda$

Hard Coefficients: $T_{i\zeta}(u)$, $T_{iJ}(u)$

M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$	M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$
$\pi^-\pi^+,\ \rho^-\pi^+,\ \pi^-\rho^+,\ \rho_\parallel^-\rho_\parallel^+$	$c_1^{(d)}+c_4^{(d)}$	0	$\pi^+K^{(*)-},\ \rho^+K^-,\ \rho_\parallel^+K_\parallel^{*-}$	0	$c_1^{(s)}+c_4^{(s)}$
$\pi^-\pi^0,\ \rho^-\pi^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0K^{(*)-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^-\rho^0,\ \rho_\parallel^-\rho_\parallel^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0K^-,\ \rho_\parallel^0K_\parallel^{*-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^0\pi^0$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^-\bar{K}^{(*)0},\ \rho^-\bar{K}^0,\ \rho_\parallel^-\bar{K}_\parallel^{*0}$	0	$-c_4^{(s)}$
$\rho^0\pi^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0\bar{K}^{(*)0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$\rho_\parallel^0\rho_\parallel^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0\bar{K}^0,\ \rho_\parallel^0\bar{K}_\parallel^{*0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$K^{(*)0}K^{(*)-},\ K^{(*)0}\bar{K}^{(*)0}$	$-c_4^{(d)}$	0	$K^{(*)-}K^{(*)+}$	0	0

similar for T_J's in terms of $b_i^{(f)}$'s

Note: have not used isospin yet

Matching

$$c_1^{(f)} = \lambda_u^{(f)}\Big(C_1+\frac{C_2}{N_c}\Big) - \lambda_t^{(f)}\frac{3}{2}\Big(C_{10}+\frac{C_9}{N_c}\Big) + \Delta c_1^{(f)}\,,$$

$$b_1^{(f)} = \lambda_u^{(f)}\Big[C_1+\Big(1-\frac{m_b}{\omega_3}\Big)\frac{C_2}{N_c}\Big] - \lambda_t^{(f)}\Big[\frac{3}{2}C_{10}+\Big(1-\frac{m_b}{\omega_3}\Big)\frac{3C_9}{2N_c}\Big] + \Delta b_1^{(f)}\,,$$

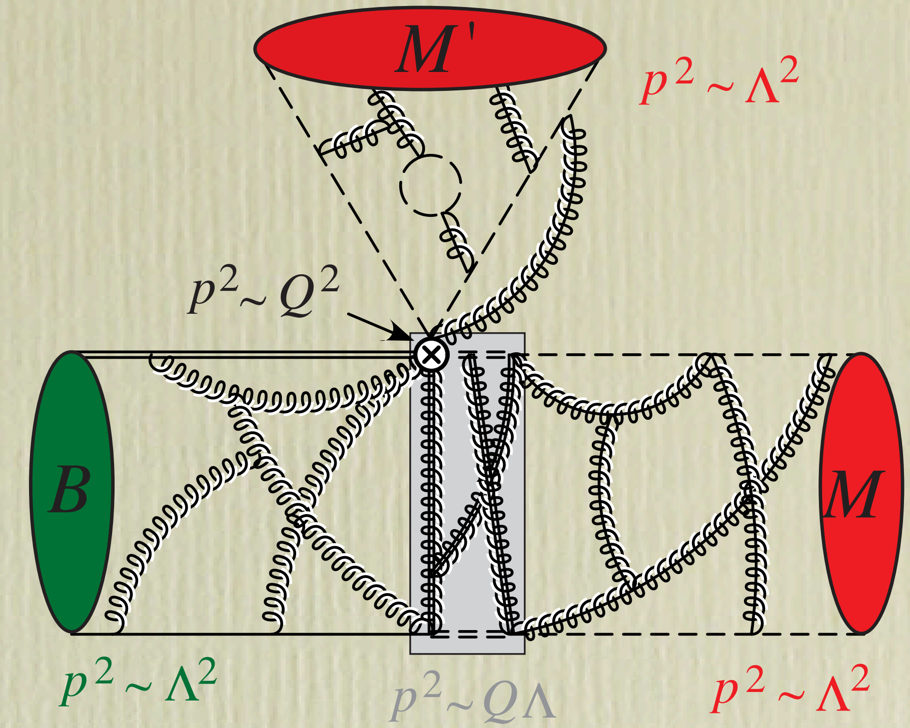
$\Delta c_i^{(f)}$ known at one-loop — Beneke et al.

$\Delta b_i^{(f)}$ known at one-loop for O1,2 — Beneke & Jager

Running

$c_i^{(f)}$ Bauer, Pirjol, Fleming, I.S.; Brodsky & Lepage

$b_i^{(f)}$ Becher, Hill, Neubert; Brodsky & Lepage



$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\left\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$$

Factorization at $\sqrt{E\Lambda}$

expansion in $\alpha_s(\sqrt{E\Lambda})$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z, x, k^+, E) \phi_M(x) \phi_B(k^+)$$

$\zeta^{BM} = ?$ (left as a form factor)

Beneke, Feldmann
Bauer, Pirjol, I.S.
Becher, Hill, Lange, Neubert

$B \to M_1 M_2$

Formalism Comments

- $\Lambda^2 \ll E\Lambda \ll E^2, m_b^2$ corrections $\sim 20\%$

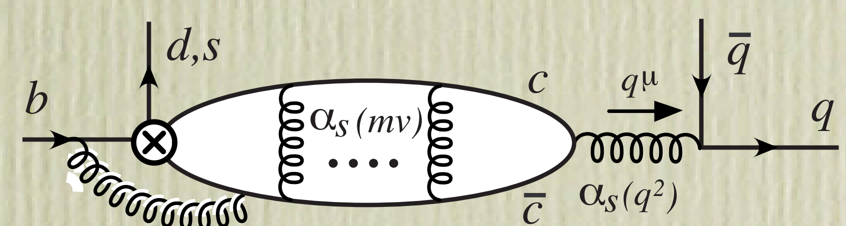
not great precision, but sufficient for large new physics signals (and improvable)

eg. Large Annihilation $C_1 \dfrac{\Lambda}{E}$

- with pert. theory at $\sqrt{E\Lambda}$ agrees with Factorization proposed by

Beneke, Buchalla, Neubert, Sachrajda

- sizeable charm loops



Ciuchini et al, Colangelo et al

long distance $A^{c\bar{c}} \sim A^{LO}\left\{ v\, \alpha_s(2m_c) \right\}$ short distance $\sim A^{LO}\left\{ \alpha_s(m_b) \right\}$

- $1/x^2$ singularity prevents further factorization of ζ^{BM}

use $k_\perp$ Factorization? pQCD

Keum, Li, Sanda, Lu et al.

(a good model for soft physics ?)

Phenomenology

I) BBNS expand in $\alpha_s(Q)$ & $\alpha_s(\sqrt{E\Lambda})$

from elsewhere input $\phi_M(x), \phi_B(k^+), \zeta^{BM}$

(eg. light-cone sum rules)

$\zeta_J^{BM} \sim \alpha_s \zeta^{BM}$

include perturbative charm & certain power corrections

II) "Charming penguins"

RGI amplitudes

fit penguin containing charm

can use factorization like I) for other terms

III) BPRS, "SCET" expand in $\alpha_s(Q)$, but keep all orders in $\alpha_s(\sqrt{E\Lambda})$

fit ζ^{BM}, ζ_J^{BM}

$\zeta^{B\pi} \sim \zeta_J^{B\pi}$

fit penguins containing charm loop using only isospin

neglect power corrections to non-penguin amplitudes

($\alpha_s(Q)$ corrections will require input)

Worth remembering:

more theory input
= less fit parameters
= more ways to test for new physics

The more results from QCD we decide are trustworthy the better the chances to find new physics

Counting parameters

<table>
<tr><th></th><th>no expn.</th><th>SU(2)</th><th>SU(3)</th><th>SCET +SU(2)</th><th>SCET +SU(3)</th></tr>
<tr><td>$B \to \pi\pi$</td><td>11</td><td>7/5</td><td rowspan="2">15/13</td><td>4</td><td rowspan="2">4</td></tr>
<tr><td>$B \to K\pi$</td><td>15</td><td>11</td><td>+5(6)</td></tr>
<tr><td>$B \to K\bar{K}$</td><td>11</td><td>11</td><td>+4/0</td><td>+3(4)</td><td>+0</td></tr>
</table>

a/b remove small $O_{8,9}$

$$\pi\pi : \quad \{\zeta^{B\pi} + \zeta_J^{B\pi}, \beta_\pi \zeta_J^{B\pi}, P_{\pi\pi}\},$$

$$K\pi : \quad \{\zeta^{B\pi} + \zeta_J^{B\pi}, \beta_{\bar{K}} \zeta_J^{B\pi}, \zeta^{B\bar{K}} + \zeta_J^{B\bar{K}}, \beta_\pi \zeta_J^{B\bar{K}}, P_{K\pi}\},$$

$$\beta_M = \int_0^1 dx \, \frac{\phi_M(x)}{3x}$$

Counting parameters

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$B \to \pi\pi$	11	7/5	15/13	4	4
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$B \to K\bar{K}$	11	11	+4/0	+3(4)	+0

use isospin to reduce errors !

	$\mathrm{Br} \times 10^6$	$A_{\mathrm{CP}} = -C$	S
$\pi^+\pi^-$	5.0 ± 0.4	0.37 ± 0.10	-0.50 ± 0.12
$\pi^0\pi^0$	1.45 ± 0.29	~~0.28 ± 0.40~~	
$\pi^+\pi^0$	5.5 ± 0.6	0.01 ± 0.06	$-$

α from $B \to \pi\pi$

Bauer, Rothstein, I.S.

Isospin + bare minimum from Λ/m_b expansion

small strong phase between two "tree" amplitudes

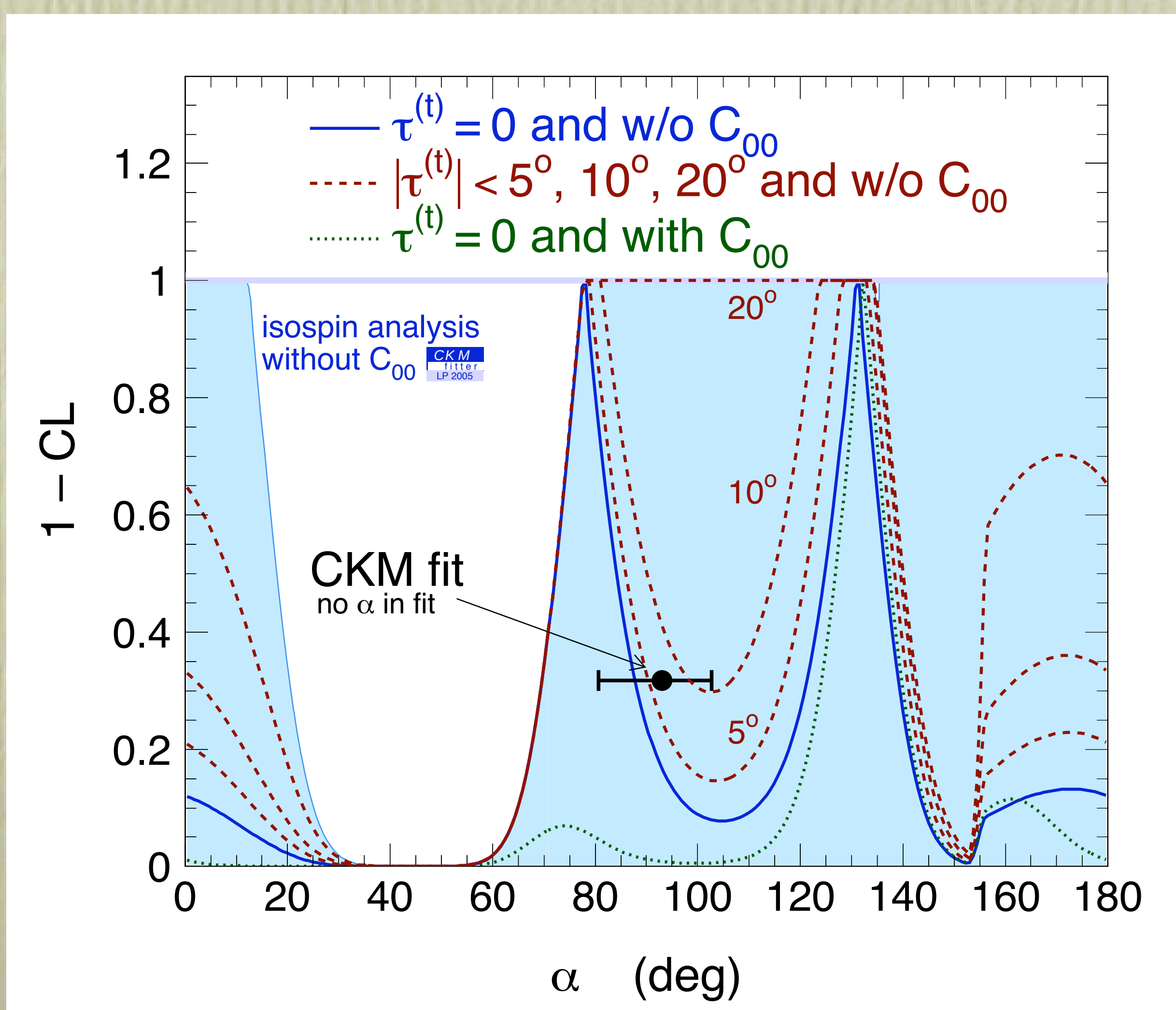
$$\mathrm{Im}\Big(\frac{C}{T}\Big) \sim \mathcal{O}\Big(\alpha_s(m_b), \frac{\Lambda}{E_\pi}\Big)$$

$$\Rightarrow \quad \gamma^{\pi\pi} = 83.0^{\circ}{}^{+7.2^\circ}_{-8.8^\circ} \pm 2^\circ$$

compare

$$\gamma^{\text{CKMfitter}}_{\text{global}} = 58.6^{\circ}{}^{+6.8^\circ}_{-5.9^\circ}\,,$$

$$\gamma^{\text{UTfit}}_{\text{global}} = 57.9^\circ \pm 7.4^\circ\,,$$



Grossman, Hoecker, Ligeti, Pirjol

Counting parameters

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Expand in $\epsilon = \underbrace{\left|\frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}}\right|}_{0.02} \frac{T}{P}\ ,\ \left|\frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}}\right| \frac{C}{P}\ ,\ \frac{P_{ew}^{(t,c)}}{P}$

$B \to K\pi$

Sum Rules

- Br sum rule:

Lipkin, many authors

$$R(\pi^0 K^-) - \frac{1}{2} R(\pi^- K^+) + R(\pi^0 K^0) = \mathcal{O}(\epsilon^2)$$

$$R(f) = \frac{\Gamma(B \to f)}{\Gamma(\bar{B}^0 \to \pi^- \bar{K}^0)}$$

$$0.094 \pm 0.073 = \mathcal{O}(\epsilon^2) = 0.03 \pm 0.02$$

no puzzle here yet

estimate from factorization in SCET

- Direct-CP sum rule:

Neubert, Gronau, Rosner

$$\Delta(\bar{K}^0\pi^0) - \frac{1}{2}\Delta(K^+\pi^-) + \Delta(K^+\pi^0) - \frac{1}{2}\Delta(\bar{K}^0\pi^-) = \mathcal{O}(\epsilon^2)$$

$$\Delta(f) = \frac{A_{CP}(f)\Gamma^{\rm CP}_{\rm avg}(f)}{\Gamma^{\rm CP}_{\rm avg}(\pi^- \bar{K}^0)}$$

$$0.07 \pm 0.08 = \mathcal{O}(\epsilon^2) = 0 \pm 0.007$$

no puzzle here yet

estimate from factorization in SCET

$$\propto \epsilon^2 \ \sin(\delta - \delta^{ew})$$

Counting parameters

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Fix: $(V_{ub} = 4.25\ 10^{-3})$ and $\langle u^{-1}\rangle_\pi \equiv 3\beta_\pi = 3.2$

Include theory errors in fit

For $\gamma = 83^o$ we find

$\zeta^{B\pi} = 0.088 \pm 0.049$

$\zeta_J^{B\pi} = 0.085 \pm 0.036$

$10^3 P_{\pi\pi} = (5.5 \pm 1.5) e^{i(151 \pm 10)}$

For $\gamma = 59^o$ we find

$\zeta^{B\pi} = 0.094 \pm 0.042$

$\zeta_J^{B\pi} = 0.100 \pm 0.027$

$10^3 P_{\pi\pi} = (2.6 \pm 1.1) e^{i(103 \pm 25)}$

Find: $\zeta^{B\pi} \sim \zeta_J^{B\pi}$

Then Predict:

$Br(\pi^0\pi^0) = (1.4 \pm .6)\ 10^{-6}$ $\quad$ $Br(\pi^0\pi^0) = (1.3 \pm .5)\ 10^{-6}$

$$Br(\pi^0\pi^0)^{\rm expt} = 1.45 \pm 0.29$$

$C(\pi^0\pi^0) = 0.49 \pm 0.26$ $\quad$ $C(\pi^0\pi^0) = 0.61 \pm 0.27$

$$C(\pi^0\pi^0)^{\rm expt} = -0.28 \pm 0.40$$

$\simeq 3$

- for $\zeta_J^{B\pi} \sim \zeta^{B\pi}$, a term $\frac{C_1}{N_c} \langle \bar{u}^{-1} \rangle_\pi \, \zeta_J^{B\pi}$ in the factorization theorem ruins color suppression and explains the rate

  if $\zeta^{B\pi} \gg \zeta_J^{B\pi}$ this Br is sensitive to power corrections (small wilson coeffs. at LO could compete with larger ones at subleading order) . ~ 0.3

- In the future: determine parameters using improved data on the $B \to \pi \ell \bar{\nu}$ form factor at low q^2 to provide a check.

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no SU(3)!

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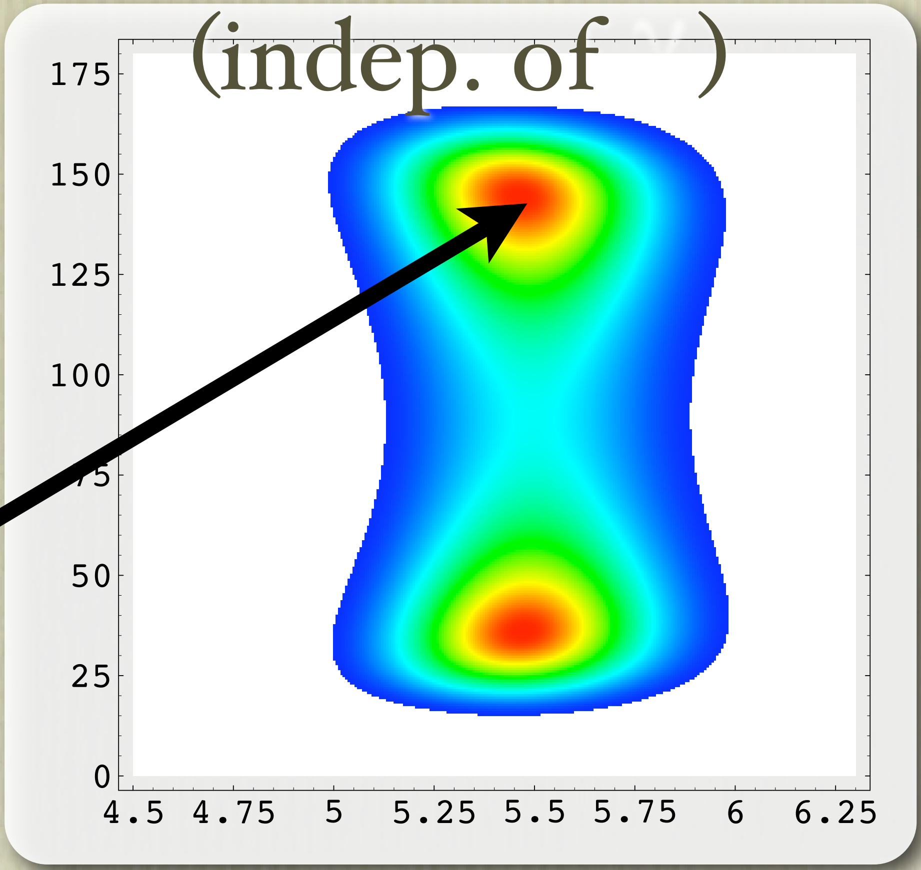
$\pi^0\pi^0$	1.45 ± 0.29	0.28 ± 0.40	
$\pi^+\pi^0$	5.5 ± 0.6	0.01 ± 0.06	–
$\pi^-\bar{K}^0$	24.1 ± 1.3	-0.02 ± 0.04	–
$\pi^0 K^-$	12.1 ± 0.8	0.04 ± 0.04	–
$\pi^+ K^-$	18.9 ± 0.7	-0.115 ± 0.018	–
$\pi^0 \bar{K}^0$	11.5 ± 1.0	-0.02 ± 0.13	0.31 ± 0.26
K^+K^-	0.06 ± 0.12		
$K^0\bar{K}^0$	0.96 ± 0.25		
$\bar{K}^0 K^-$	1.2 ± 0.3		–

Combined $\pi\pi$ & $K\pi$

penguin phase

(indep. of)



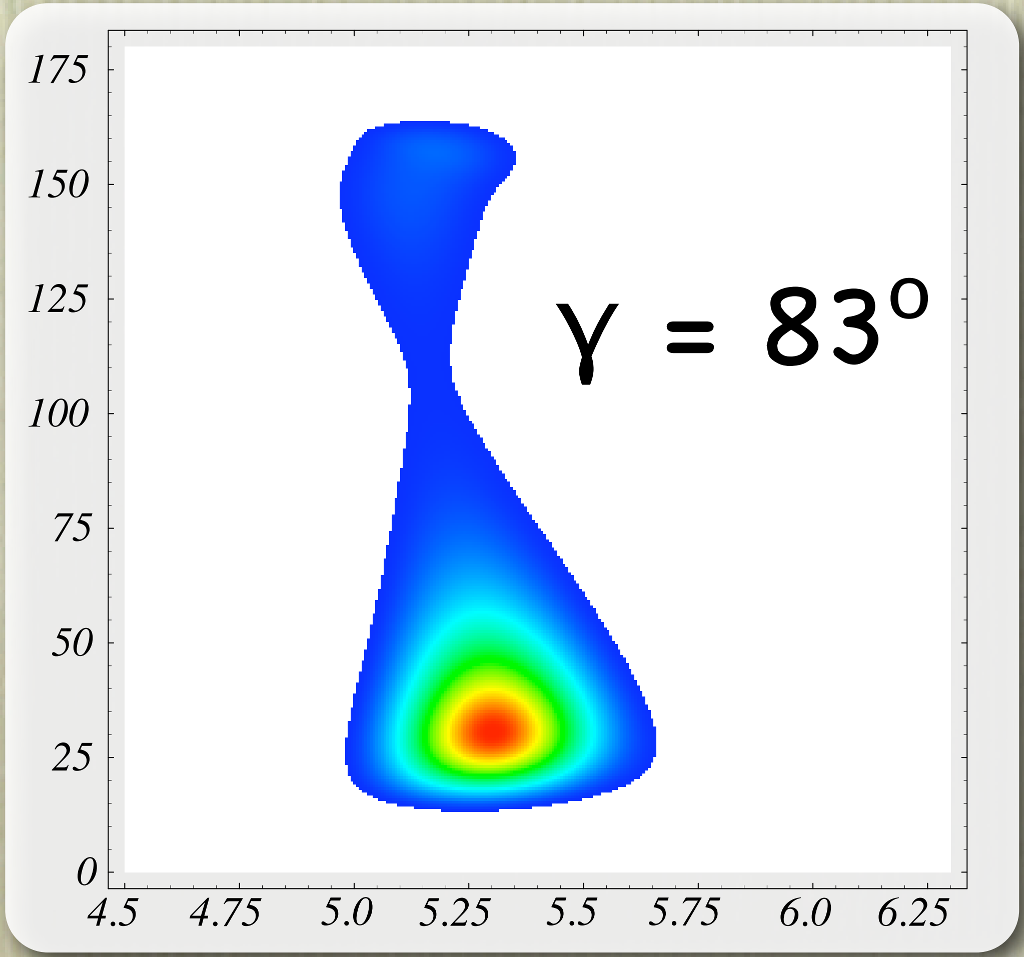
penguin amplitude

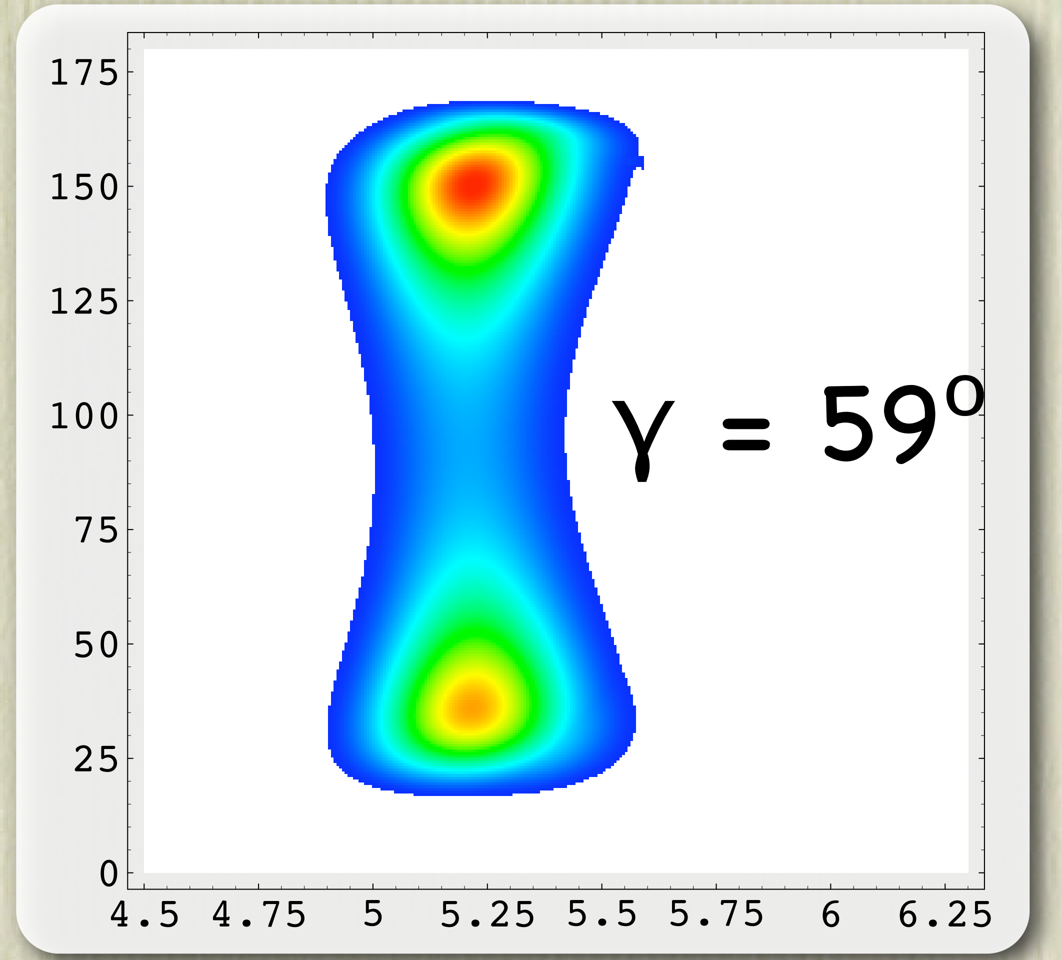
Include Br($K^0\pi^-$), Acp($K^+\pi^-$)

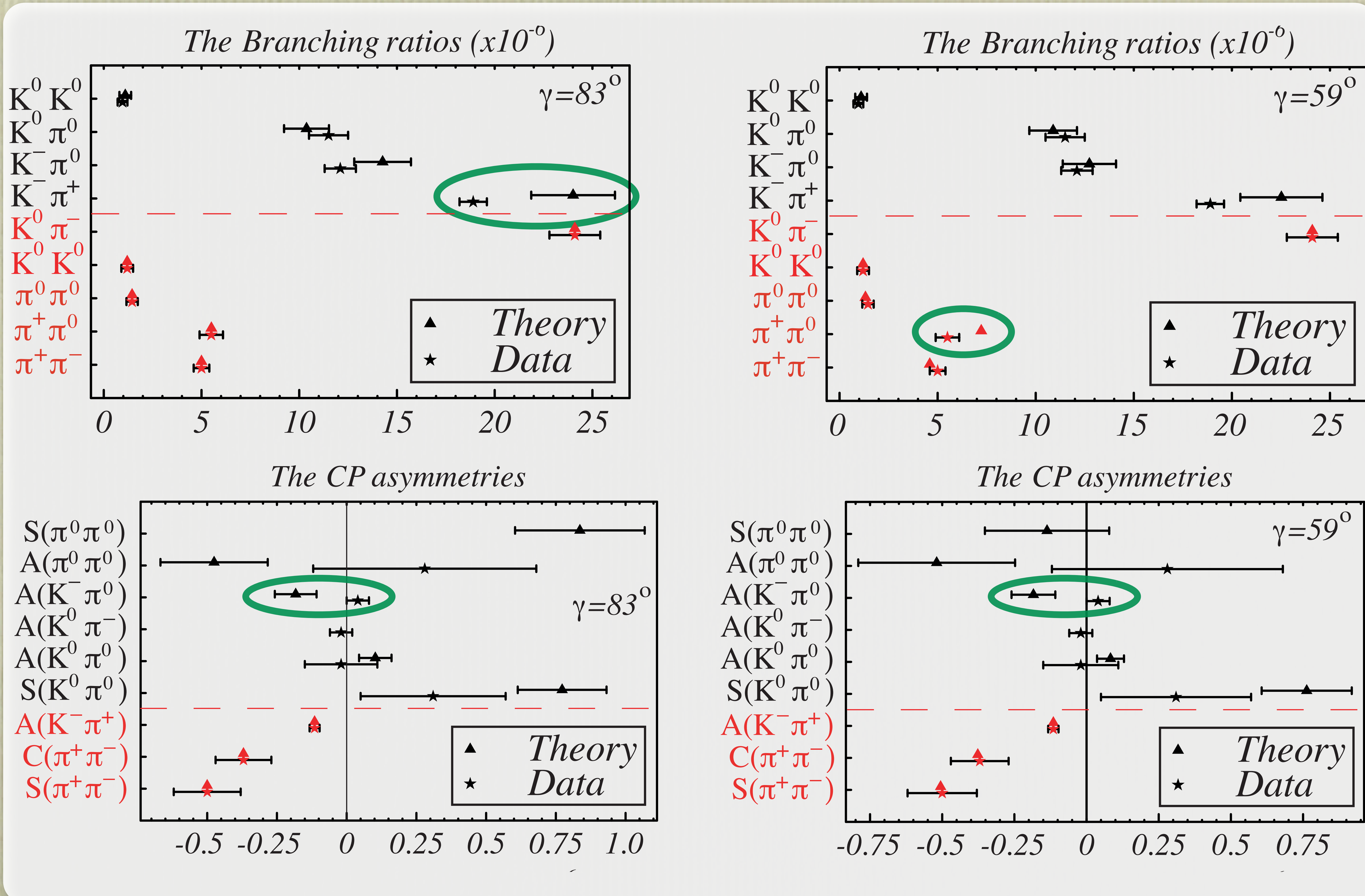
SU(3) preferred if $\gamma = 83$

$10^3 P_{\pi\pi} = (5.5 \pm 1.5) e^{i(151 \pm 10)}$

Include Br($K^+\pi^-$)





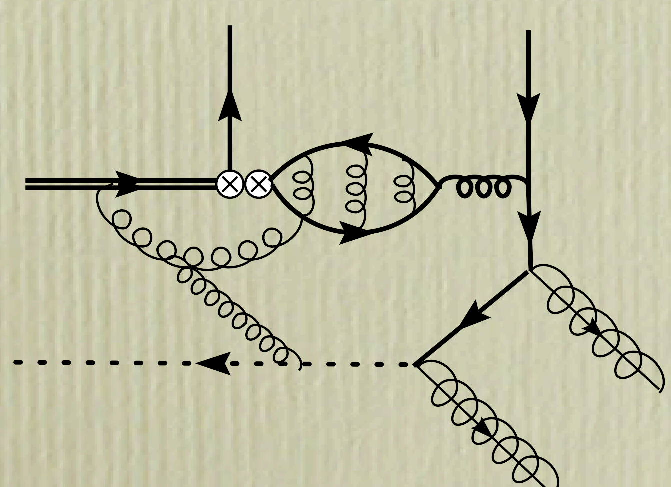

The Branching ratios (x10^{-6})
γ=83°
$K^0 K^0$
$K^0 \pi^0$
$K^- \pi^0$
$K^- \pi^+$
$K^0 \pi^-$
$K^0 K^0$
$\pi^0 \pi^0$
$\pi^+ \pi^0$
$\pi^+ \pi^-$
Theory
Data
0
5
10
15
20
25
The Branching ratios (x10^{-6})
γ=59°
$K^0 K^0$
$K^0 \pi^0$
$K^- \pi^0$
$K^- \pi^+$
$K^0 \pi^-$
$K^0 K^0$
$\pi^0 \pi^0$
$\pi^+ \pi^0$
$\pi^+ \pi^-$
Theory
Data
0
5
10
15
20
25
The CP asymmetries
γ=83°
$S(\pi^0 \pi^0)$
$A(\pi^0 \pi^0)$
$A(K^- \pi^0)$
$A(K^0 \pi^-)$
$A(K^0 \pi^0)$
$S(K^0 \pi^0)$
$A(K^- \pi^+)$
$C(\pi^+ \pi^-)$
$S(\pi^+ \pi^-)$
Theory
Data
-0.5
-0.25
0
0.25
0.5
0.75
1.0
The CP asymmetries
γ=59°
$S(\pi^0 \pi^0)$
$A(\pi^0 \pi^0)$
$A(K^- \pi^0)$
$A(K^0 \pi^-)$
$A(K^0 \pi^0)$
$S(K^0 \pi^0)$
$A(K^- \pi^+)$
$C(\pi^+ \pi^-)$
$S(\pi^+ \pi^-)$
Theory
Data
-0.75
-0.5
-0.25
0
0.25
0.5
0.75

Counting parameters

	no expn.	SU(2)	SU(3)	SCET +SU(2)	SCET +SU(3)
$B \to \pi\pi$	11	7/5	15/13	4	4
$B \to K\pi$	15	11		+5(6)	
$B \to K\bar{K}$	11	11	+4/0	+3(4)	+0

Extension to isosinglets

$\pi\eta, \eta\eta, K\eta', \ldots$



Williamson & Zupan

+4

(2 solutions)

Predictions
(4 param. fit)

$\gamma = 59^o$

Branching Fraction
Direct CP Asymmetry

Mode	Exp.	Theory I	Theory II
$B^- \to \pi^-\eta$	4.3 ± 0.5 $(S = 1.3)$	$4.9 \pm 1.7 \pm 1.0 \pm 0.5$	$5.0 \pm 1.7 \pm 1.2 \pm 0.4$
	-0.11 ± 0.08	$0.05 \pm 0.19 \pm 0.21 \pm 0.05$	$0.37 \pm 0.19 \pm 0.21 \pm 0.05$
$B^- \to \pi^-\eta'$	2.53 ± 0.79 $(S = 1.5)$	$2.4 \pm 1.2 \pm 0.2 \pm 0.4$	$2.8 \pm 1.2 \pm 0.3 \pm 0.3$
	0.14 ± 0.15	$0.21 \pm 0.12 \pm 0.10 \pm 0.14$	$0.02 \pm 0.10 \pm 0.04 \pm 0.15$
$\bar{B}^0 \to \pi^0\eta$	–	$0.88 \pm 0.54 \pm 0.06 \pm 0.42$	$0.68 \pm 0.46 \pm 0.03 \pm 0.41$
	–	$0.03 \pm 0.10 \pm 0.12 \pm 0.05$	$-0.07 \pm 0.16 \pm 0.04 \pm 0.90$
$\bar{B}^0 \to \pi^0\eta'$	–	$2.3 \pm 0.8 \pm 0.3 \pm 2.7$	$1.3 \pm 0.5 \pm 0.1 \pm 0.3$
	–	$-0.24 \pm 0.10 \pm 0.19 \pm 0.24$	–
$\bar{B}^0 \to \eta\eta$	–	$0.69 \pm 0.38 \pm 0.13 \pm 0.58$	$1.0 \pm 0.4 \pm 0.3 \pm 1.4$
	–	$-0.09 \pm 0.24 \pm 0.21 \pm 0.04$	$0.48 \pm 0.22 \pm 0.20 \pm 0.13$
$\bar{B}^0 \to \eta\eta'$	–	$1.0 \pm 0.5 \pm 0.1 \pm 1.5$	$2.2 \pm 0.7 \pm 0.6 \pm 5.4$
	–	–	$0.70 \pm 0.13 \pm 0.20 \pm 0.04$
$\bar{B}^0 \to \eta'\eta'$	–	$0.57 \pm 0.23 \pm 0.03 \pm 0.69$	$1.2 \pm 0.4 \pm 0.3 \pm 3.7$
	–	–	$0.60 \pm 0.11 \pm 0.22 \pm 0.29$
$\bar{B}^0 \to \bar{K}^0\eta'$	63.2 ± 4.9 $(S = 1.5)$	$63.2 \pm 24.7 \pm 4.2 \pm 8.1$	$62.2 \pm 23.7 \pm 5.5 \pm 7.2$
	0.07 ± 0.10 $(S = 1.5)$	$0.011 \pm 0.006 \pm 0.012 \pm 0.002$	$-0.027 \pm 0.007 \pm 0.008 \pm 0.005$
$\bar{B}^0 \to \bar{K}^0\eta$	< 1.9	$2.4 \pm 4.4 \pm 0.2 \pm 0.3$	$2.3 \pm 4.4 \pm 0.2 \pm 0.5$
	–	$0.21 \pm 0.20 \pm 0.04 \pm 0.03$	$-0.18 \pm 0.22 \pm 0.06 \pm 0.04$
$B^- \to K^-\eta'$	69.4 ± 2.7	$69.5 \pm 27.0 \pm 4.3 \pm 7.7$	$69.3 \pm 26.0 \pm 7.1 \pm 6.3$
	0.031 ± 0.021	$-0.010 \pm 0.006 \pm 0.007 \pm 0.005$	$0.007 \pm 0.005 \pm 0.002 \pm 0.009$
$B^- \to K^-\eta$	2.5 ± 0.3	$2.7 \pm 4.8 \pm 0.4 \pm 0.3$	$2.3 \pm 4.5 \pm 0.4 \pm 0.3$
	-0.33 ± 0.17 $(S = 1.4)$	$0.33 \pm 0.30 \pm 0.07 \pm 0.03$	$-0.33 \pm 0.39 \pm 0.10 \pm 0.04$

errors: su3, 1/mb, fit

Counting parameters VP, VV modes

	no expn.	SU(2)	SU(3)	SCET +SU(2)	SCET +SU(3)
$B \to \pi\pi$	11	7/5	15/13	4	4
$B \to K\pi$	15	11		+5(6)	
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SCET+SU(2) counting for:

$B \to \rho_{\parallel}\rho_{\parallel}$	4
$B \to K^*\pi$	+5 (6)
$B \to K\rho$	+2 (6)
$B \to K^*_{\parallel}\rho_{\parallel}$	+2 (6)
$B \to \rho\pi$	+4 (8)
$\vdots$	$\vdots$

Rough Analysis

Fix: (V_{ub} = 4.25 10^{-3})

For $\gamma = 83^o$ I find

$\zeta^{B\rho}+\zeta_J^{B\rho} = 0.27 \pm 0.02$

$\beta_\rho \zeta_J^{B\rho} = 0.09$

$10^3 P_{\rho\rho} = (7.6) e^{i(-3^o)}$

For $\gamma = 59^o$ I find

$\zeta^{B\rho}+\zeta_J^{B\rho} = 0.29 \pm 0.02$

$\beta_\rho \zeta_J^{B\rho} = 0.07$

$10^3 P_{\rho\rho} = (2.9) e^{i(8^o)}$

$\zeta^{B\rho} \gg \zeta_J^{B\rho}$? closer to BBNS counting

Then Predict:

$Br(\rho^0\rho^0) = (2.8)\ 10^{-6}$ $Br(\rho^0\rho^0) = (1.9)\ 10^{-6}$

at isospin bound

$$Br(\rho^0\rho^0)^{\rm expt} < (1.1) \times 10^{-6}$$

for $\langle u^{-1} \rangle_\rho / 3 \equiv \beta_\rho \simeq 0.8\ \beta_\pi$ ratio $\dfrac{\zeta_J^{B\rho}}{\zeta_J^{B\pi}}$ agrees with $\alpha_s(\sqrt{E\Lambda})$ perturbation theory

Counting parameters VP, VV modes

	no expn.	SU(2)	SU(3)	SCET +SU(2)	SCET +SU(3)
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SCET+SU(2) counting for:

$B \to \rho_{||}\rho_{||}$ 4

$B \to K^*\pi$ +5 (6)

$B \to K\rho$ +2 (6)

$B \to K^*_{||}\rho_{||}$ +2 (6)

} # observables similar to $K\pi$

$B \to \rho\pi$ +4 (8)

$\vdots$ $\vdots$

can make predictions to test factorization or determine γ

Counting parameters VP, VV modes

<table>
<tr><th></th><th>no expn.</th><th>SU(2)</th><th>SU(3)</th><th>SCET +SU(2)</th><th>SCET +SU(3)</th></tr>
<tr><td>$B \to \pi\pi$</td><td>11</td><td>7/5</td><td rowspan="2">15/13</td><td>4</td><td rowspan="2">4</td></tr>
<tr><td>$B \to K\pi$</td><td>15</td><td>11</td><td>+5(6)</td></tr>
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</table>

SCET+SU(2) counting for:

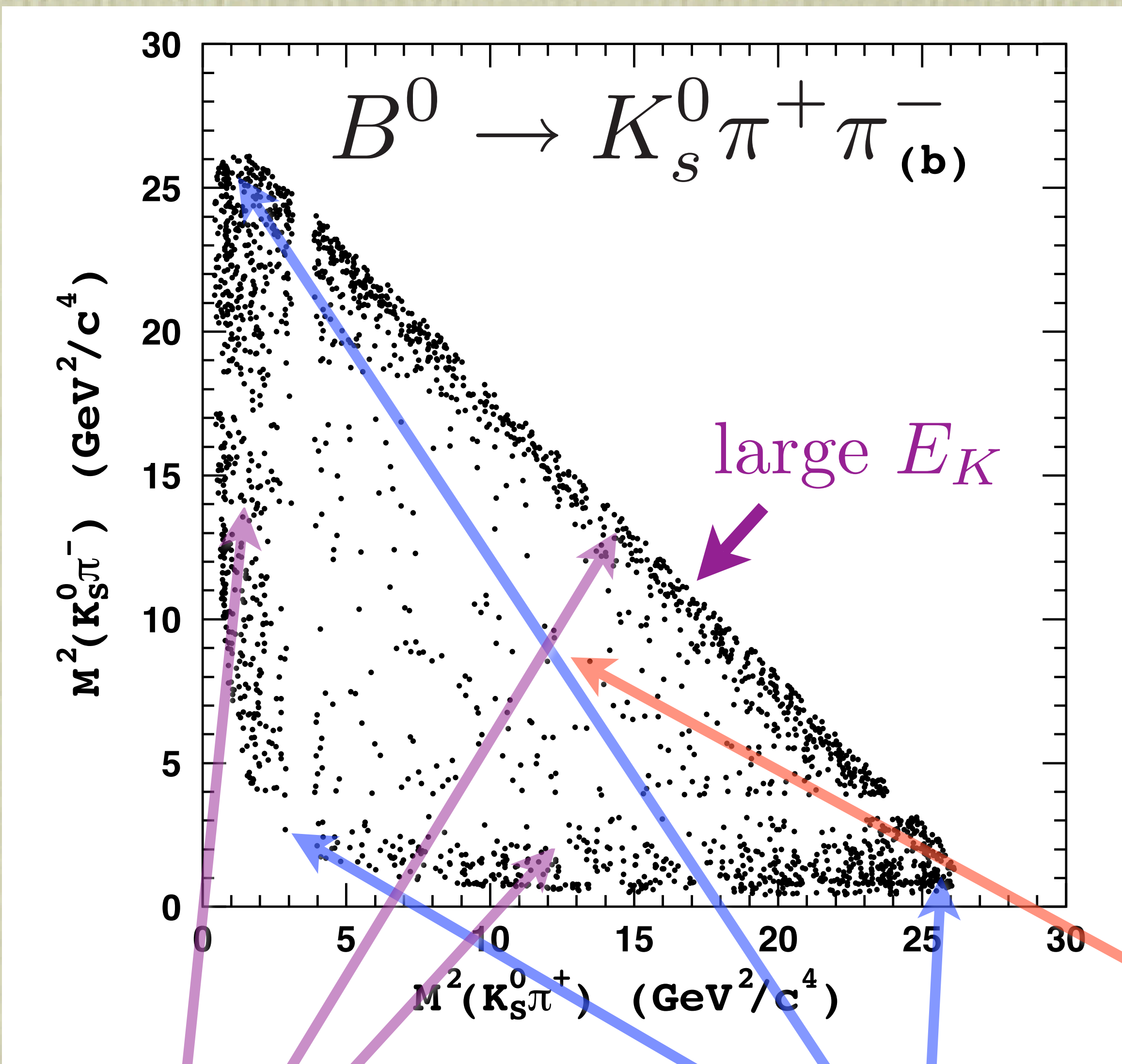
$B \to \rho_{\parallel}\rho_{\parallel}$	4
$B \to K^*\pi$	+5 (6)
$B \to K\rho$	+2 (6)
$B \to K^*_{\parallel}\rho_{\parallel}$	+2 (6)
$B \to \rho\pi$	+4 (8)
$\vdots$	$\vdots$

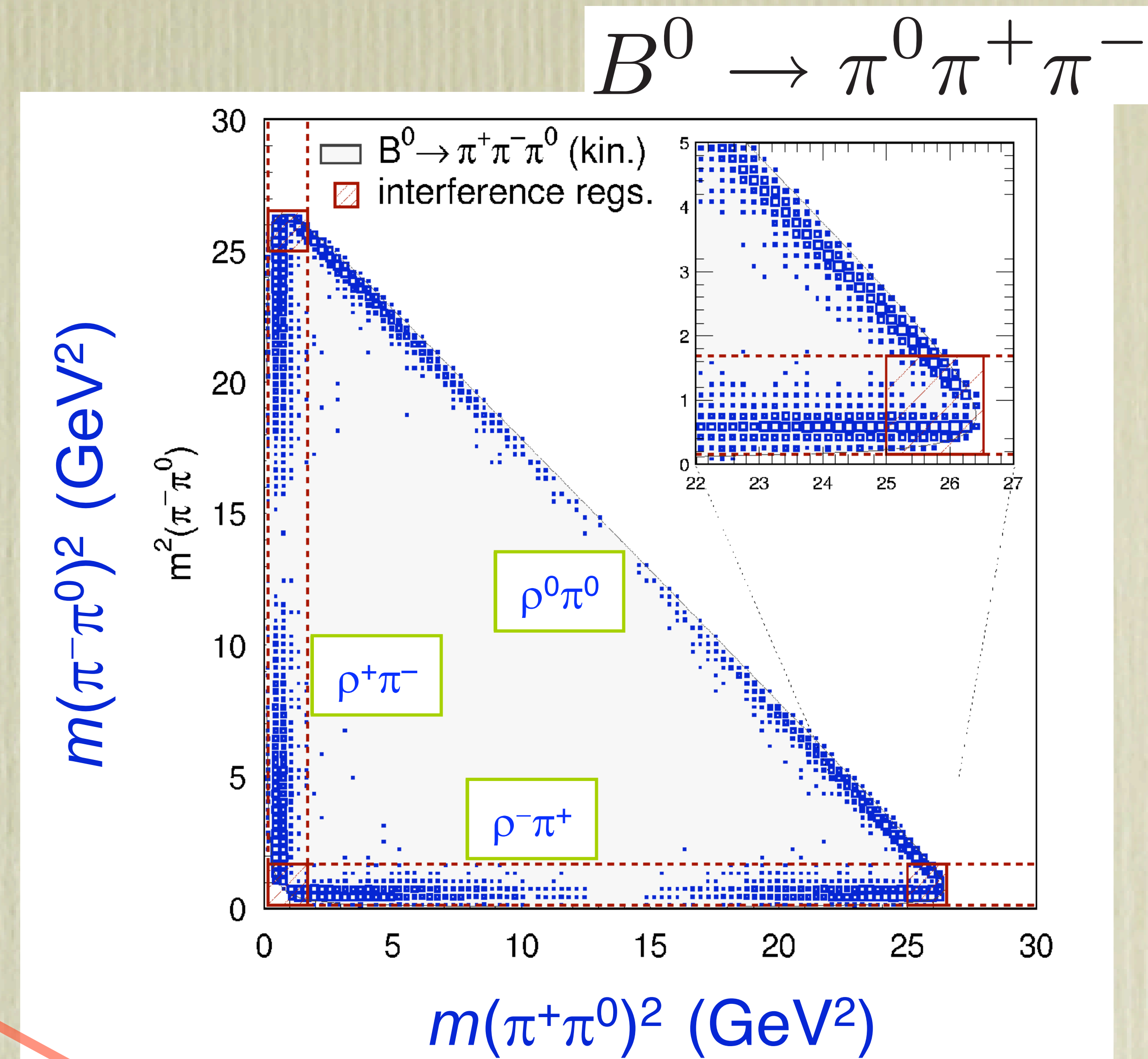
} can make predictions to test factorization or determine γ

Three -body Decays with Factorization

(Results derived back of the envelope, while at this meeting)

Assume $Q = m_b/3 \gg \Lambda_{\rm QCD}$





$B \to M^1_n M^2_n M^3_{\bar n}$ $\qquad$ $B \to M^1_n M^2_{\bar n} M^3_s$ $\qquad$ $B \to M^1_n M^2_{\bar n} M^3_{n'}$

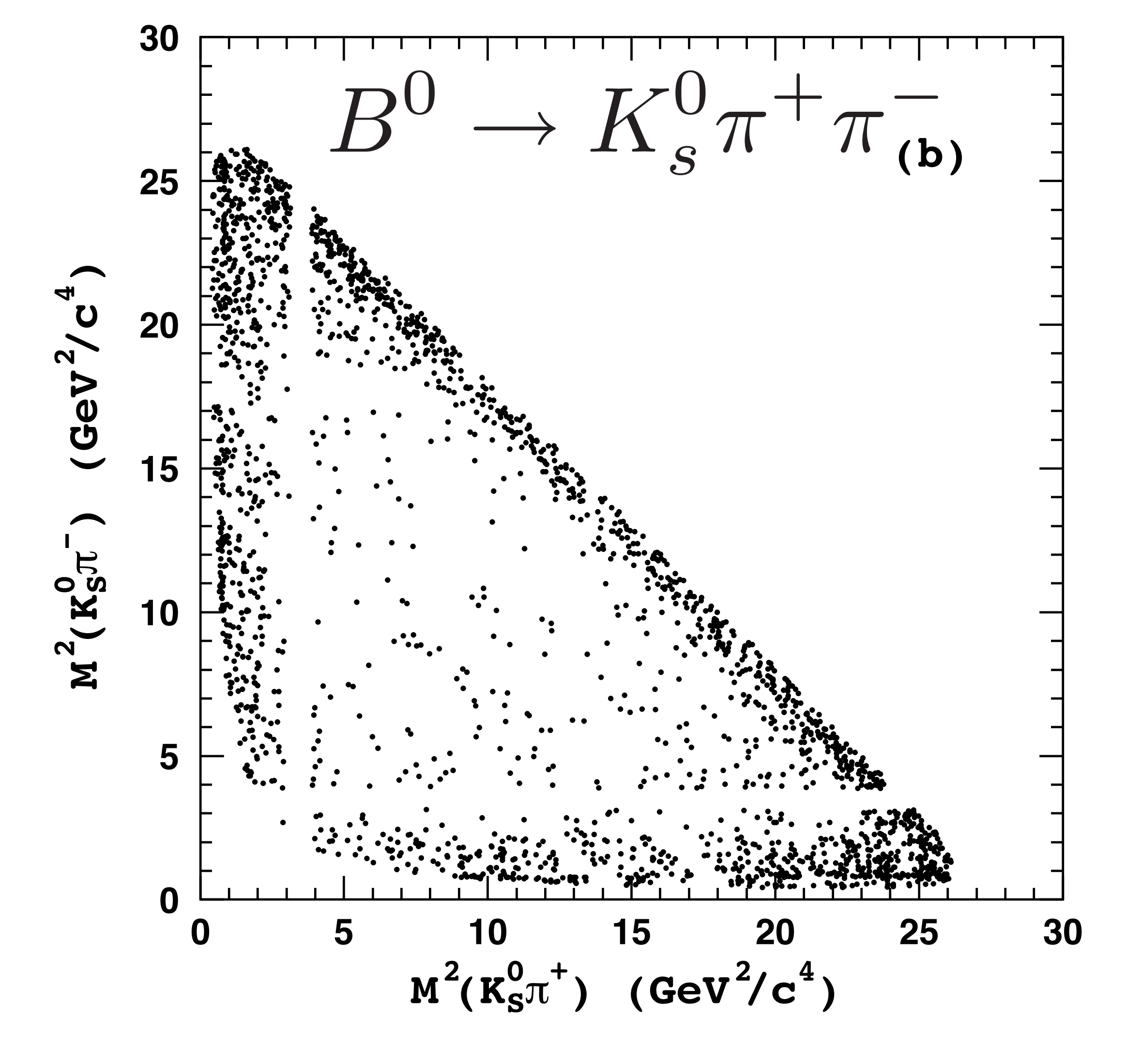
$A \sim 1$ for all $m^2_{12} \le Q\Lambda$ $\qquad$ $A \sim 1$ $\qquad$ $A \sim 1/Q^2$

$B \to M_n^1 M_n^2 M_{\bar{n}}^3$

- same operators as

$$B \to M_n^1 M_{\bar{n}}^2$$

- different state



two-meson
distn. function

Factorization:

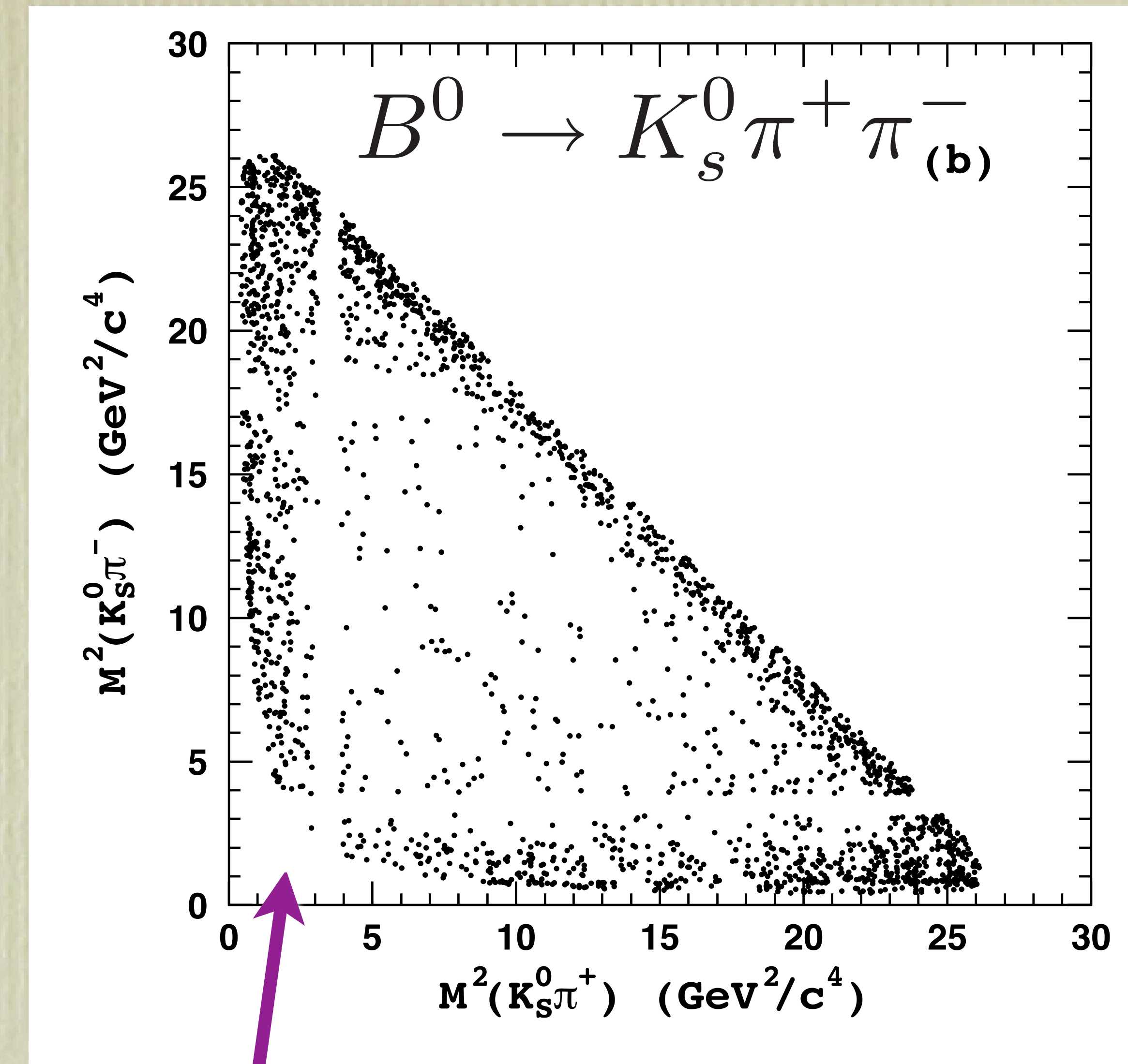
$$A = \zeta^{BM_1M_2}\ T \otimes \phi^{M_3} + \zeta^{BM_3}\ T \otimes \phi^{M_1M_2} + (\zeta_J\ \text{terms})$$

$$B \to M_n^1 M_{\bar{n}}^2 M_s^3$$

- same operators as

$$B \to M_n^1 M_{\bar{n}}^2$$

- different state



small E_K

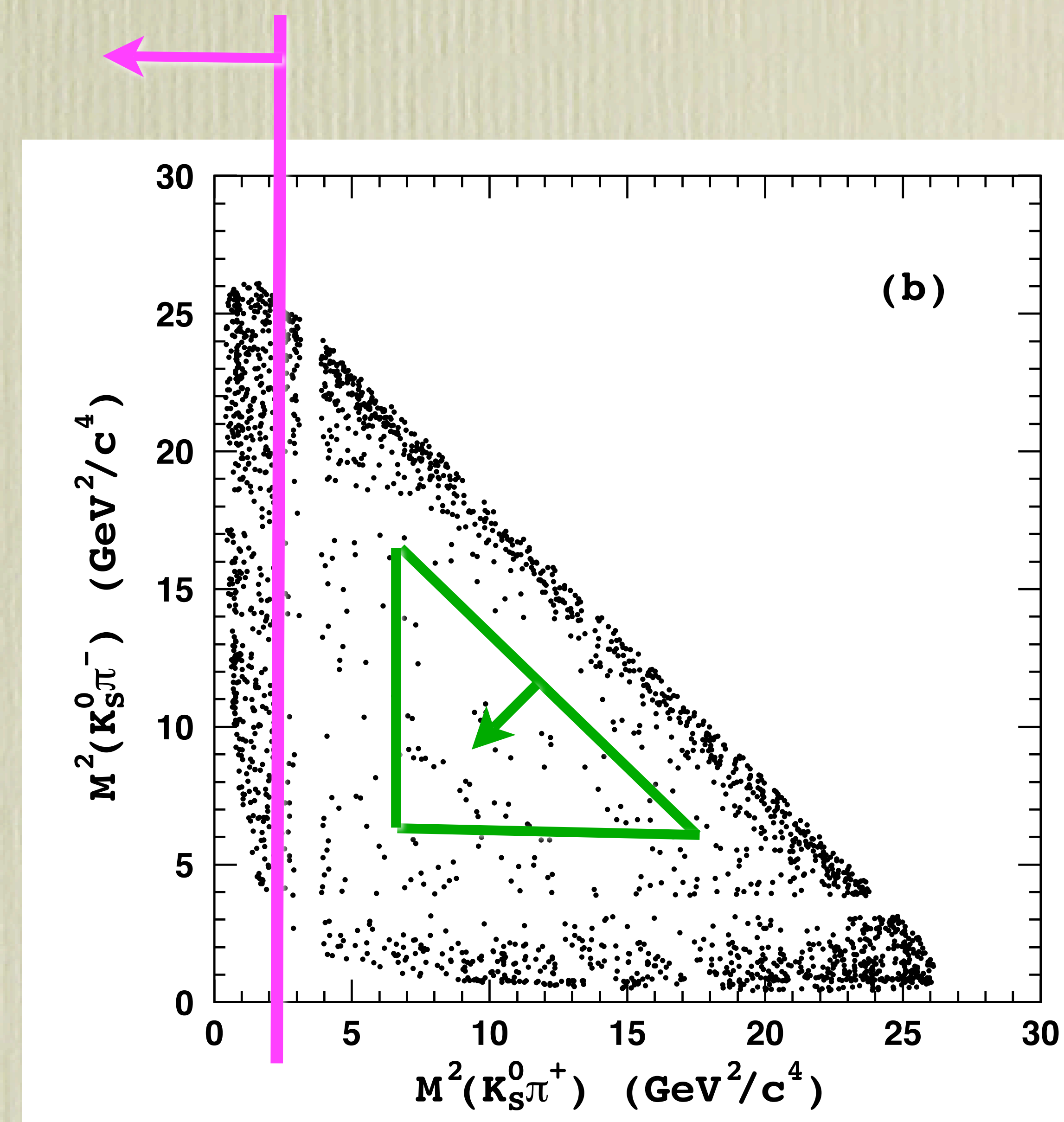
strange quark must be collinear at LO !

Factorization:

$$A = \zeta^{BM_1M_3}\ T \otimes \phi^{M_2} + \zeta^{BM_2M_3}\ T \otimes \phi^{M_1} + (\zeta_J \text{ terms})$$

Thoughts

- factorization will provide additional strong phase information
- can use $\gamma^*\gamma \to M_1 M_2$ for $\phi^{M_1 M_2}$
- can use $B \to D M_1 M_2$ for $\phi^{M_1 M_2}$
- can use $B \to M_1 M_2 e\bar{\nu}$ for $\zeta^{B M_1 M_2}$



- enhanced SU(3) predictions, eg. can use SU(3) on $\phi^{M_1 M_2}$

- From theory point of view: simpler to predict amplitudes with cuts

The END